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Large-N limit of non-commutative gauge theories

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Abstract: Using the correspondence between gauge theories and string theory in curved backgrounds, we investigate aspects of the large-N limit of non-commutative gauge theories by considering gravity solutions with B fields. We argue that the total number of physical degrees of freedom at any given scale coincides with the commutative case. We then compute a two-point correlation function involving momentum components in the directions of the B-field. In the infrared regime it reproduces the usual behavior of the commutative gauge theory (i.e. of the form $k^4 \log k^2$). In the UV regime, we find that the two-point function decays exponentially with the momentum. A calculation of Wilson lines suggests that strings cannot be localized near the boundary. We also find string configurations that are localized in a finite region of the radial direction. These are worldsheet instantons.

Keywords: Duality in Gauge Field Theories, Black Holes in String Theory, Brane Dynamics in Gauge Theories.

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Contents

1. Introduction

Gauge theories on non-commutative spaces can arise in certain limits of string theory [1, 2, 3, 4]. Specifically, one considers a system of Dp-branes with a constant NS B along their worldvolume directions. In general, there are open strings and closed strings coupled together, but it is possible to take a low energy limit — rescaling some parameters of the theory, like the metric along the worldvolume directions and the string coupling constant — in such a way that the closed strings decouple and the resulting action for the open string modes is the one corresponding to a non-commutative gauge theory [1, 4].

In this paper we consider gravity solutions that are related to Dp branes with B fields. In the spirit of [5, 6], the near horizon region of these gravity solutions should describe the large-N limit of non-commutative gauge theories. We will see that the limit we have to take in order to isolate the near horizon region is the same as the limit that is taken from the field theory point of view. These gravity solutions reduce to the usual Dp brane solutions very close to the horizon; this corresponds to the fact that the field theory reduces to the commutative field theory at long distances. In all cases the dilaton goes to zero at the boundary faster than in the commutative counterparts. Close to the boundary the solution is quite different from the usual Dp brane solution and it typically contains a varying $H = dB$ NS field as well as some RR field strengths. The presence of a B field induces a D$(p-2)$ charge density. In fact, we can gauge away B in the bulk at the expense of introducing a U(1) field strength on the brane theory. So some of the gravity solutions are essentially the ones found in [7, 8].

Our discussion will mostly concentrate on solutions describing D3 branes with B fields. We will see that correlation functions between operators with vanishing momentum along the directions with non-zero B field give the same correlation functions as in the case with zero B-field. This is in agreement with the arguments in [9], which indicate that planar diagrams depend on the non-commutativity parameter only through the external momenta. Correlators with dependence on the momenta along the B directions are quite different. In particular, we find that the equation for small fluctuations of a certain graviton mode is formally the same as the wave equation in the full D3-brane geometry. The renormalization factors necessary to define the correlators depend on the momenta. This renders two-point functions rather ambiguous, since one can substract any function of the external momenta. (The ratio of three point functions to two point functions should be unambiguous). We discuss a prescription to define correlation functions, and calculate the two-point function corresponding to the components of energy-momentum tensor of the gauge theory. At low energies, the correlator reproduces the usual $k^4 \log k^2$ behavior of the standard commutative gauge theory. At high energies, we find an exponential fall off with the momentum.

By the holographic principle, one can determine the number of degrees of freedom by computing the area of surfaces at fixed r, and relating them to the number of degrees of freedom of the field theory with an energy cutoff of order r. The number of degrees of freedom, computed in this fashion, is the same as in the commutative case, though there seems to be redistribution of them, in a sense that will be explained.

The supergravity solution in Minkowski space with a B field in the time direction is also given. This corresponds to a D3-brane with electric and magnetic U(1) worldvolume fields. Other generalizations are discussed in section 6. We give gravity solutions corresponding to M5 branes in the presence of a constant $C_{\mu\nu\rho}$ field — a theory whose DLCQ version was studied in [10] — and we construct solutions for D1-D5 systems with B fields. These include a solution where the B field is along the worldvolume of the D1 brane, and the case where the B field is on a torus inside the 5-brane world volume.

While this paper was being written we received the paper [11] which has some overlap with section 2.

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2. Construction of the solutions

2.1 D3-brane in constant B field

In this section we obtain the solution of a D3-brane in a constant NS B field background. All solutions with non vanishing B fields in this section preserve 16 supersymmetries. First we consider the case where there is only one component B_{23} different from zero. A simple way to obtain the solution is to perform a T-duality along x_3. This gives a smeared D2-brane on a tilted torus. It is easy to write down this solution. Then we T-dualize back on x_3, using the T-duality rules for RR backgrounds derived in [12]. The solution in string metric is

$$
\begin{aligned}
ds^2_{str} &= f^{-1/2}\left[-dx_0^2+dx_1^2+h(dx_2^2+dx_3^2)\right]+f^{1/2}(dr^2+r^2d\Omega_5^2)\,,\\
f &= 1+\frac{\alpha'^2R^4}{r^4}\,, & h^{-1} &= \sin^2\theta f^{-1}+\cos^2\theta\,,\\
B_{23} &= \frac{\sin\theta}{\cos\theta}f^{-1}h\,, & e^{2\phi} &= g^2h\,,\\
F_{01r} &= \frac{1}{g}\sin\theta\,\partial_r f^{-1}\,, & F_{0123r} &= \frac{1}{g}\cos\theta h\partial_r f^{-1}\,.
\end{aligned}
\tag{2.1}
$$

The asymptotic value of the B field is $B_{23}^{\infty}=\tan\theta$. The parameter R is defined by

$$
\cos\theta\,R^4 = 4\pi gN\,,
\tag{2.2}
$$

where N is the number of D3 branes, and $g\equiv g_\infty$ is the asymptotic value of the coupling constant. It is possible to gauge away a constant B-field. However, this introduces a constant flux for the worldvolume gauge field, since under $B\to B+d\Lambda$, $A\to A-\Lambda$. In fact, performing this gauge transformation we obtain a solution which is the same as (2.1) except that the value of the B field is shifted by $B\to B-B^\infty$. In that form, it is easy to see that the solution has D3- and D1-brane charges. This solution was found in [7, 8], and it represents D1-branes dissolved in D3-branes.

In euclidean space, one can also construct a solution with $B_{01}=\tan\theta'$, $B_{23}=\tan\theta$ in a similar way. The solution in string metric is given by

$$
\begin{aligned}
ds^2_{str} &= f^{-1/2}\Big[h'(dx_0^2+dx_1^2)+h(dx_2^2+dx_3^2)\Big]+f^{1/2}\left[dr^2+r^2d\Omega_5^2\right],\\
f &= 1+\frac{\alpha'^2R^4}{r^4}\,, & e^{2\phi} &= g^2hh'\,,\\
h^{-1} &= \sin^2\theta f^{-1}+\cos^2\theta\,, & h'^{-1} &= \sin^2\theta' f^{-1}+\cos^2\theta'\,,\\
B_{01} &= \frac{\sin\theta'}{\cos\theta'}f^{-1}h'\,, & B_{23} &= \frac{\sin\theta}{\cos\theta}f^{-1}h\,,\\
A_{01} &= i\frac{1}{g}\sin\theta\cos\theta'\,h'f^{-1}\,, & \chi &= i\frac{1}{g}\sin\theta\sin\theta' f^{-1}\,,\\
F_{0123r} &= i\frac{1}{g}\cos\theta\cos\theta' hh'\partial_r f^{-1}\,. & A_{23} &= i\frac{1}{g}\sin\theta'\cos\theta\,hf^{-1}\,,
\end{aligned}
\tag{2.3}
$$

The parameter R is given by $R^4=4\pi gN(\cos\theta\cos\theta')^{-1}$.

There is an analogous solution with lorentzian signature which is obtained by a Wick rotation $x_0 \to ix_0$ and $\theta' \to i\theta'$ (so that $\cos\theta' \to \cosh\theta'$, $\sin\theta' \to i\sinh\theta'$). As a result, the imaginary factors in the gauge fields disappear. The lorentzian solution can also be obtained from eleven dimensions by starting with a stack of M2-branes (with world-volume coordinates x_0, x_1, x_2) smeared in two direcions x_3, x_4. One redefines coordinates as follows:

$$\begin{aligned} x_4 &= \tilde{x}_4 \cos\alpha + (\tilde{x}_2 \cos\theta + \tilde{x}_3 \sin\theta)\sin\alpha\,, \\ x_2 &= -\tilde{x}_4 \sin\alpha + (\tilde{x}_2 \cos\theta + \tilde{x}_3 \sin\theta)\cos\alpha\,, \\ x_3 &= -\tilde{x}_2 \sin\theta + \tilde{x}_3 \cos\theta\,. \end{aligned} \tag{2.4}$$

Next, we make dimensional reduction in $\tilde{x}_4$ and a T-duality transformation in the $\tilde{x}_3$-direction. We find

$$\begin{aligned} ds^2_{str} &= f^{-1/2}\left[h'(-dx_0^2 + dx_1^2) + h(dx_2^2 + dx_3^2)\right] + f^{1/2}\left[dr^2 + r^2 d\Omega_5^2\right], \\ f &= 1 + \cos^2\alpha \frac{\alpha'^2 R_0^4}{r^4}\,, \qquad h = \frac{f}{G}\,, \qquad h' = \frac{f}{H}\,, \\ H &= 1 + \frac{\alpha'^2 R_0^4}{r^4}\,, \qquad G = 1 + \cos^2\alpha\cos^2\theta \frac{\alpha'^2 R_0^4}{r^4}\,, \\ e^{2\phi} &= g^2 h h'\,. \end{aligned} \tag{2.5}$$

The function H is the harmonic function appearing in the original M2 brane metric. The solution is indeed the Wick rotated version of (2.3), with the identification of parameters: $R_0^4 = R^4 \cosh^2\theta'$, $\cos^2\alpha = 1/\cosh^2\theta'$. The particular case $\theta = 0$ appeared in [13]. In this case the background (2.5) reduces to the solution which is obtained by applying S-duality transformation to the solution (2.1).

2.2 Decoupling limit

The above solutions are asymptotic to flat space for $r \to \infty$ and they have a horizon at $r = 0$. Very near $r = 0$ the solutions look like $AdS_5 \times S^5$. The throat region connecting these two contains non-zero NS and RR B fields. If we take the standard low energy limit, keeping all other parameters constant we just get the usual AdS solution. On the boundary we have the usual $\mathcal{N} = 4$ Yang-Mills theory. In order to obtain non-commutative Yang-Mills we should also take the B field to infinity. The supergravity solution itself tells us what limit we should take to decouple the asymptotic region while still keeping the region of the spacetime where B fields vary.

For the solution (2.1) the rescaling of the parameters should be the following

$$\begin{aligned} \alpha' &\to 0\,, \qquad \tan\theta = \frac{\tilde{b}}{\alpha'}\,, \\ x_{0,1} &= \tilde{x}_{0,1}\,, \qquad x_{2,3} = \frac{\alpha'}{\tilde{b}}\tilde{x}_{2,3}\,, \\ r &= \alpha' R^2 u\,, \qquad g = \alpha'\tilde{g}\,, \end{aligned} \tag{2.6}$$

where $\tilde{b}$, u, $\tilde{g}$, $\tilde{x}_\mu$ stay fixed. This scaling of parameters is precisely the same as the scaling that was found in [4]. The factor of $\tilde{b}$ in the second line of (2.6) is introduced just for later convenience.

$$\begin{aligned}
ds^2_{str} &= \alpha' R^2 \left[u^2(-d\tilde{x}_0^2 + d\tilde{x}_1^2) + u^2 \hat{h}(d\tilde{x}_2^2 + d\tilde{x}_3^2) + \frac{du^2}{u^2} + d\Omega_5^2 \right], \\
\hat{h} &= \frac{1}{1+a^4u^4}, \qquad\qquad a^2 = \tilde{b}R^2, \\
\tilde{B}_{23} &= B_\infty \frac{a^4u^4}{1+a^4u^4}, \qquad\qquad B_\infty = \alpha' \frac{1}{\tilde{b}} = \alpha' \frac{R^2}{a^2}, \\
e^{2\phi} &= \hat{g}^2 \hat{h}, \\
A_{01} &= \alpha' \frac{\tilde{b}}{\hat{g}} u^4 R^4, \qquad\qquad \tilde{F}_{0123u} = \alpha'^2 \frac{\hat{h}}{\hat{g}} \partial_u (u^4 R^4),
\end{aligned} \tag{2.7}$$

where $\hat{g} = \tilde{g}\tilde{b}$ is the value of the string coupling in the IR. Now we have $R^4 = 4\pi\hat{g}N$. Notice the B and A fields are defined with respect to the new coordinates and they include some rescalings implied by (2.6). We propose that (2.7) is the dual gravity description of non-commutative Yang-Mills. This solution reduces to the $AdS_5 \times S^5$ solution for small u, which corresponds to the IR regime of the gauge theory. This is consistent with the expectation that non-commutative Yang-Mills reduces to ordinary Yang-Mills theory at long distances. The parameter a has the dimension of length, and the solution starts deviating from the $AdS_5 \times S^5$ solution at $u \sim 1/a$, i.e. at a distance scale of the order $a = R\sqrt{\tilde{b}}$. For large $\hat{g}N \sim R^4$, this is greater than the naively expected[1] distance scale of $L \sim \sqrt{\tilde{b}}$. This can be interpreted as due to strong interactions, which seem to render visible the effects of non-commutativity at longer distances than naively expected. The solution has a boundary at $u = \infty$. As we approach the boundary, the physical size of the $2,3$ direction shrinks (in string units), since $\hat{h} \sim 1/u^4$ for large u. All curvature invariants in string metric remain bounded as we approach the boundary. The reason for this is that the string metric near the boundary has a scaling isometry: $x_{0,1} \to \lambda^{-1}x_{0,1}$, $u \to \lambda u$, $x_{2,3} \to \lambda x_{2,3}$. Consequently, curvature invariants of the metric near $u = \infty$ (obtained by $\hat{h} \to 1/(au)^4$) cannot depend on u, for $u \to \infty$. However, if we were to compactify the $2,3$ directions we would get a singularity since some winding modes would become light. This singularity cannot be avoided by going to the T-dual description. In fact, T-duality transformations along x_2, x_3 lead to the same background (2.1) with the exchange $\cos\theta \leftrightarrow \sin\theta$. In the case that the B field is rational the singularity can be removed by performing an appropriate $\mathrm{SL}(2,\mathbb{Z})$ T-duality transformation, as explained in [1]. These decoupled solutions still have 16 supersymmetries, there is no supersymmetry enhancement since the theory is not conformal.

[1]From the limiting form of the two point function of the coordinate of the boundary of an open string we find that $[\tilde{x}^2, \tilde{x}^3] \sim \tilde{b}$ [1, 4].

The near-horizon background (2.7) can also be obtained from eleven dimensions by starting from the standard M2 brane solution smeared in two directions x_3, x_4, making dimensional reduction in a null circle $x^+ = x_4 + x_0$, and T-duality in x_3. This is related to the observation of [1] that M theory compactified on a null circle with non-zero C_{+ij} leads to non-commutative geometry. Thus the solution (2.7) has also the eleven-dimensional interpretation of an M2 brane with an infinite boost in a transverse direction.

Let us now consider the decoupling limit of the geometry (2.3). The rescaling of the parameters is basically the same as (2.6) except that now one should also rescale the $0,1$ coordinates and the string coupling in a slightly different way,

$$g = \alpha'^2 \tilde{g}\,, \qquad x_{0,1} = \frac{\alpha'}{\tilde{b}'}\tilde{x}_{0,1}\,. \tag{2.8}$$

We introduce parameters a, a' with dimension of length by $a^2 = \tilde{b}R^2,\ a'^2 = \tilde{b}'R^2$. We get (after relabelling $\tilde{x}_i \to x_i$ of the world-volume coordinates)

$$\begin{aligned}
ds^2_{str} &= \alpha' R^2\Big[u^2[\hat{h}'(dx_0^2 + dx_1^2) + \hat{h}(dx_2^2 + dx_3^2)] + \frac{du^2}{u^2} + d\Omega_5^2\Big]\,, \\
e^{2\phi} &= \hat{g}^2\hat{h}\hat{h}'\,, \qquad \hat{h} = \frac{1}{1 + a^4u^4}\,, \qquad \hat{h}' = \frac{1}{1 + a'^4u^4}\,, \\
B_{01} &= \alpha' R^2\frac{a'^2u^4}{1 + a'^4u^4}\,, \qquad B_{23} = \alpha' R^2\frac{a^2u^4}{1 + a^4u^4}\,, \\
A_{01} &= i\alpha'\frac{\tilde{b}}{\hat{g}}\hat{h}'R^4u^4\,, \qquad A_{23} = i\alpha'\frac{\tilde{b}'}{\hat{g}}\hat{h}R^4u^4\,, \\
F_{0123u} &= i\alpha'^2\frac{\hat{h}\hat{h}'}{g}\partial_u(R^4u^4)\,, \qquad \chi = i\frac{\tilde{b}\tilde{b}'}{\hat{g}}R^4u^4\,.
\end{aligned} \tag{2.9}$$

where again $\hat{g} = \tilde{b}\tilde{b}'\tilde{g}$ is the the value of the string coupling in the IR. This solution reduces to the standard $AdS_5 \times S^5$ solution for small u. The effects of non-commutativity appear at the largest of the two scales $a = R\sqrt{\tilde{b}}$ or $a' = R\sqrt{\tilde{b}'}$. For large u the physical size of the spatial directions decreases, but all curvature invariants in string metric are bounded. In fact, the string metric for large u is again $AdS_5 \times S^5$. Although the point $u = \infty$ may look like the "horizon" of AdS, the other fields are different from the fields of the usual $AdS_5 \times S^5$ solution. In particular, the dilaton e^ϕ goes to zero at large u. This implies that fluctuations close to the boundary are suppressed (that is why we call it "boundary"). This boundary is similar in spirit to the case of the NS fivebrane. There also we find that the metric in string units remains constant as we move away from the horizon. For the NS fivebrane the dilaton goes to zero and this suppresses interactions as we approach the boundary. The fact that the dilaton goes to zero also freezes the asymptotic geometry, which is the physical property that we expect from a boundary. A completely analogous situation arises in the near horizon geometry of D-instantons. There again the dilaton goes to zero and the physical size in string metric goes to zero. In fact, the D-instanton boundary looks like the origin of R^{10} except that the string coupling is going to zero.

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This similarity is not a coincidence, in fact the behavior of the solution (2.9) close to $u \sim \infty$ is the same as the behavior of a D-instanton smeared in the 0123 directions. If we compute the area in Planck units of a surface of constant u we see that it increases as u^4. This is the same behavior that one finds in the pure commutative Yang-Mills.

The decoupling limit for the metric (2.5) with lorentzian signature has a different form, since the limit must be taken in a different way. As it is clear from the eleven-dimensional origin as a stack of rotated M2 branes, the appropriate limit now amounts to dropping the "1" in the harmonic function H. The string coupling e^ϕ becomes strong at large u in this case, and it is more appropriate to go to the S-dual metric. The scaling of parameters is as follows:

$$\begin{aligned} \cos\alpha &= \frac{\alpha'}{\tilde{b}'}\,, \qquad & \cos\alpha\cos\theta &= \frac{\alpha'}{\tilde{b}}\,, \\ x_{0,1} &= \tilde{x}_{0,1}\,, \qquad & x_{2,3} &= \frac{\alpha'}{\tilde{b}}\tilde{x}_{2,3}\,, \\ r &= \alpha' R^2 u\,, \qquad & g &= \frac{\alpha'}{\tilde{b}'}\hat{g}\,. \end{aligned} \tag{2.10}$$

We get

$$\begin{aligned} ds^2 &= \alpha' R_0^2 \left(\frac{\hat{h}'}{\hat{h}}\right)^{1/2} \left[u^2\left[-d\tilde{x}_0^2 + d\tilde{x}_1^2 + \hat{h}(d\tilde{x}_2^2 + d\tilde{x}_3^2)\right] + \frac{du^2}{u^2} + d\Omega_5^2\right], \\ e^{2\phi} &= \hat{g}^2\frac{\hat{h}'^2}{\hat{h}}\,, \end{aligned} \tag{2.11}$$

where $\hat{h}$ and $\hat{h}'$ were defined in (2.9) (with $a^2 = \tilde{b}R_0^2$, $a'^2 = \tilde{b}'R_0^2$). Since $\cos\theta = \tilde{b}'/\tilde{b} \leq 1$, the solution exists for $\tilde{b}' \leq \tilde{b}$ (for $\tilde{b}' > \tilde{b}$ some gauge fields proportional to $\sin\theta$, i.e. B_{01}, A_{23} and χ, become imaginary). The above near-horizon background contains as particular cases (2.7) ($\tilde{b} = \tilde{b}'$) and its S-dual version ($\tilde{b}' = 0$).

3. Generalization to non-extremal metrics

3.1 Non-commutative gauge theory at finite temperature

A B-field can be similarly introduced in the non-extremal D3-brane background by the same U-duality transformations that were made above for the extremal case. The resulting metric is simply obtained by multiplying the g_{00} component by $(1-u_h^4/u^4)$, and the g_{uu} component by $(1-u_h^4/u^4)^{-1}$. In particular, the non-extremal version of the solution (2.7) in the Einstein frame $ds_E^2 = e^{-\phi/2}ds_{str}^2$ is given by

$$\begin{aligned} ds_E^2 = R^2(1+a^4u^4)^{1/4}\Bigg(u^2\left[-\left(1-\frac{u_h^4}{u^4}\right)dx_0^2 + dx_1^2 + \frac{1}{(1+a^4u^4)}(dx_2^2+dx_3^2)\right] + \\ + \frac{du^2}{u^2(1-\frac{u_h^4}{u^4})} + d\Omega_5^2\Bigg). \end{aligned} \tag{3.1}$$

String theory on this non-extremal background should provide a dual description of non-commutative Yang-Mills theory at finite temperature.

An interesting question concerns the number of degrees of freedom of non-commutative gauge theories. Because of the non-commutative nature of the space-time coordinates, one may expect that in the ultraviolet regime there could be a reduction of the degrees of freedom relative to the usual gauge theories. The number of gauge-invariant degrees of freedom is reflected on the dependence of the free energy with the temperature. Let us denote by S_0, E_0 and F_0 the entropy, energy and free energy of the black D3-brane with $B_{23} = 0$ ($a = 0$). The Hawking temperature of the metric (3.1) is clearly the same as the temperature of the $a = 0$ metric, since the u, x_0 part of the metric is only modified by a conformal factor. The entropy S for the metric (3.1) is determined by the area of the event horizon times the volume of the D3-brane, that is, $S = \frac{V_8}{V_8^0} S_0$, where V_8 is the volume of the $(x_1, x_2, x_3, \Omega_5)$ space, and V_8^0 the corresponding volume for the metric with $a = 0$. From eq. (3.1), it follows that $V_8 = V_8^0$, and therefore $S = S_0$. By the first law of thermodynamics, $dE = T_H dS$, one can anticipate that the energy is also unchanged. In fact, it is easy to see that this is the case by calculating the mass directly. Thus $F = E - T_H S = F_0$. The fact that all thermodynamic quantities are the same as in ordinary gauge theories is indicating that the number of degrees of freedom is also the same. The same conclusion applies for the other solutions discussed in this paper. This is consistent with the arguments of [9], saying that all large-N computations with no external lines should give the same result, which is what we find.

Although the total number of degrees of freedom remains unchanged relative to the usual case as the energy scale is varied, there seems to be a redistribution of them. As $u \to \infty$, a contraction of the volume of the torus x_2, x_3 is compensated by an expansion of the volume of the sphere. This means that momentum modes become heavier while angular modes become lighter.

3.2 Non-commutative models with $\mathcal{N} = 0$ supersymmetries

Let us now consider the system (2.9) with B_{01} and B_{23} in the particular case $a = a'$. The generalization to the non-extremal case is easy if one starts with the usual non-extremal M2 brane metric, and make dimensional reduction and T-duality as described in (2.4), and finally the Wick rotation to euclidean signature. We then take the decoupling limit as before.[2] The near-horizon, non-extremal metric in the Einstein frame is given by

$$ds_E^2 = \alpha' \frac{R^2 a^2}{\sqrt{\hat{g}}} \left(\tilde{f}^{-1/2} \left[\left(1 - \frac{u_h^4}{u^4} \right) dx_0^2 + dx_1^2 + dx_2^2 + dx_3^2 \right] + \tilde{f}^{1/2} \left[\frac{du^2}{1 - \frac{u_h^4}{u^4}} + u^2 d\Omega_5^2 \right] \right),$$
$$e^{2\phi} = \hat{g}^2 (1 + a^4 u^4)^{-2}, \qquad \tilde{f} = 1 + \frac{1}{a^4 u^4}. \tag{3.2}$$

[2] As explained above, the decoupling limit on the euclidean solution is not equivalent to the decoupling limit of the lorentzian solution (2.11).

(We have rescaled $x_i \to a^2 x_i$, $i = 0, \ldots, 3$). Remarkably, the metric (3.2) coincides with the euclidean metric of the black D3-brane. The full background is of course not equivalent, since other fields are also varying and we have written the dilaton as an example. The identification of parameters with the standard non-extremal solution of [14] is as follows: $a = 1/r_-$, $u_h^4 = r_+^4 - r_-^4$, where r_-, r_+ are the inner and outer horizons in coordinates $r^4 = u^4 + r_-^4$. The coordinate x_0 is periodic and describes a circle of radius ρ_0, given by

$$\rho_0 = \frac{1}{2\pi T_H} = \frac{r_+^2}{2}(r_+^4 - r_-^4)^{-1/4} = \frac{1}{2u_h}\sqrt{u_h^4 + \frac{1}{a^4}}\,. \tag{3.3}$$

The D3-brane charge of the usual [14] black D3-brane solution is related to $r_\pm$ by $r_+^2 r_-^2 = 4\pi g N \alpha'^2$. In the present case, the charge N (which is in $R^4 = 4\pi \hat{g} N$) is not related to r_+, r_-; now $r_- = 1/a$ and r_+ is a function of a and the radius ρ_0. String theory on the background (3.2) should be related to the euclidean non-commutative SU(N) gauge theory with $B_{01} = B_{23}$ compactified, with antiperiodic boundary conditions, on the circle parametrized by x_0.

The usual black D3-brane metric [14] has rather non-trivial thermodynamic properties,[3] and one can expect that this will translate into some interesting effect in the gauge theory dual to (3.2).

The parameter u_h is given in terms of ρ_0 and a by (3.3), i.e. $u_h^2 = 2\rho_0^2 \pm \sqrt{4\rho_0^4 - 1/a^4}$. There is a minimum radius $\rho_0^2 = \rho_{\min}^2 = 1/(2a^2)$, below which we cannot find smooth solutions of this kind (in terms of the original coordinates x_i with dimension of length, obtained by $x_i \to x_i/a^2$, one has $\rho_{\min} = a/\sqrt{2}$). For every ρ_0 such that $\rho_0 \geq \rho_{\min}$, there are two possible values of u_h. The one which is connected to the AdS_5 solution is that of smaller u_h. The branch with greater u_h is unstable. The reason is that it is essentially the same as a non-extremal Schwarzschild black hole in 7 dimensions.

The antiperiodic boundary condition gives rise to tree-level masses of $O(1/\rho_0)$ for all fermions and supersymmetry is completely broken to $\mathcal{N} = 0$. In the present case, the radius cannot be taken to zero, since as explained above the metric cannot be smooth at $u = u_h$ for any $\rho_0 < \rho_{\min}$. It is worth noting that the spectrum of the Laplace operator corresponding to the Einstein-frame metric (3.2) is continuous. The reason is that there are plane waves at infinity, where the metric is flat. This suggests that the mass spectrum of physical fluctuations is continuous (modulo possible subtleties about boundary conditions due to the varying dilaton and other fields). Clearly, more investigation on this model is needed.

[3] At a certain temperature $T_{\rm cr}$ (corresponding to $r_+^4/r_-^4 = \sqrt{3}$ or $M/M_{\rm ext} = 7/2\sqrt{3}$) the specific heat becomes infinity, and above that temperature it becomes negative, whereas, for the solution with $\tilde{f} \to r_-^4/u^4$, the specific heat is constant and positive.

4. Correlation functions

Let us first consider correlation functions in the case that only $B_{23} \neq 0$. If we consider fields that do not depend on $x_{2,3}$ then we can easily see that their correlators give the same result as in $AdS_5 \times S^5$ in the same situation, i.e. zero momentum in the 2,3 directions. The reason is that we obtained the solution by a U-duality transformation that involved the $2, 3$ directions. Since this is a continous symmetry of the gravity solution — regardless of whether the coordinates are compact or not — any given boundary conditions for the fields can be translated into boundary conditions in $AdS_5 \times S^5$; one can then calculate the perturbed supergravity solution, which by asumption will not depend on x_2, x_3, and T-dualize it back to obtain the correlation functions.

Non-trivial correlations will involve situations where the fields depend on x_2, x_3. Now both B_{23} and B_{01} can be nonzero. General fluctuations of the near-horizon background (2.7) or (2.9) will be coupled in a complicated system of differential equations. A simple equation is obeyed by the graviton fluctuation h_{01}. This is associated to the energy-momentum tensor component T_{01} of the Yang-Mills theory. Let us set all other fluctuations to zero and define $\varphi = g^{00} h_{01}$. One can check that all equations of motion are satisfied provided

$$\frac{e^{2\phi}}{\sqrt{g}} \partial_\mu \sqrt{g} e^{-2\phi} g^{\mu\nu} \partial_\nu \varphi = 0\,, \qquad \partial_0 \varphi = \partial_1 \varphi = 0\,, \tag{4.1}$$

where $g_{\mu\nu}$ is the string-frame metric (2.7) or (2.9). That is, the equations of motion are satisfied for fluctuations which are independent of x_0, x_1 and obey the *scalar* Laplace equation. In principle one could consider a situation where the fields depend also on x_0, x_1 but the equations might be more complicated. We will thus consider the gauge theory correlator $\langle \mathcal{O}(k) \mathcal{O}(-k) \rangle$, where $\mathcal{O}(k)$ is the Fourier transform of $T_{01}(x)$.

Correlation functions in the gauge theory are obtained by the usual prescription [15, 16]

$$\left\langle \exp\left[\int d^4k \varphi_0(k) \mathcal{O}(k) \right] \right\rangle = \exp\left[-S_{\mathrm{sugra}}[\varphi(k,u)] \right]\,, \qquad i = 0,1,2,3\,, \tag{4.2}$$

where $\varphi(k,u)$ is the unique solution with $\varphi(k,u) \to \varphi_0(k)$ at some boundary at $u = \Lambda$. We work in momentum space because operators in the field theory are defined most naturally in momentum space. The equation of motion for φ in the background (2.7) then becomes

$$\frac{1}{u^5} \partial_u u^5 \partial_u \varphi - k^2 \left(\frac{1}{u^4} + a^4 \right) \varphi = 0\,, \qquad k \equiv |k| = \sqrt{k_2^2 + k_3^2}\,. \tag{4.3}$$

For $u \cong 0$, this reduces to the Laplace equation in the $AdS_5 \times S^5$ background. If $k^2 \neq 0$, the behavior of the solutions at large u is different: in the AdS_5 case, one has $\varphi(u) \sim Au^{-4} + B$, whereas in the case (4.3) the solution behaves asymptotically as $\varphi(u) \sim u^{-5/2} e^{\pm k a^2 u}$.

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Interestingly, equation (4.3) is the same as the equation for a dilaton fluctuation in the extremal D3-brane metric, with the replacement $-k^2 \to \omega^2 R^4$, and $a = 1/R$. This differential equation was solved in [17] in terms of Mathieu functions. Indeed, by the following change of variable,

$$u^2 = \frac{1}{a^2} e^{-2z}\,, \qquad \varphi(z) = e^{2z}\psi(z)\,, \tag{4.4}$$

the equation (4.3) becomes

$$\left(\frac{\partial^2}{\partial z^2} - 2k^2a^2\cosh(2z) - 4\right)\psi(z) = 0\,, \tag{4.5}$$

which is the Mathieu differential equation with $z \to iz$.

Near $z = \infty$ (or $u \sim 0$), the solution which is well-behaved is of the form

$$\varphi(z) \cong e^{2z} K_2(ke^z) \to 0\,, \qquad \text{for } z \to \infty\,. \tag{4.6}$$

For $z \to -\infty$ (or $u \to \infty$), the solution behaves as

$$\varphi(z) \cong e^{5z/2}\left(\exp\left[kae^{-z}\right] + B(ka)\ \exp\left[-kae^{-z}\right]\right). \tag{4.7}$$

The case analyzed in [17] can be obtained by taking $z \to z + i\pi/2$; this implies that the coefficient B in our case is the same as the reflection coefficient in their situation. The correlation functions must be renormalized. The most reasonable way to do this is to impose the boundary condition

$$\varphi(k,\Lambda) = \frac{1}{\Lambda^{5/2}} e^{ka^2\Lambda}\varphi_0(k)\,, \qquad \Lambda \to \infty\,, \tag{4.8}$$

where $u = \Lambda$ is the cutoff surface where we impose the boundary conditions. This condition ensures that the solution in the interior, at a fixed value of u, remains finite as we take $\Lambda \to \infty$, which means that the effects of inserting the operator remain finite as we take the cutoff to infinity.

We evaluate the action integrating by parts and using (4.6) and (4.7). We find a boundary term

$$\begin{aligned} S &= \frac{1}{2}\int d^2k\varphi(k,u)u^5\partial_u\varphi(u,k)|_{u=\Lambda} \\ &= \int d^2k\left[\operatorname{div}(\Lambda,k,a) - 2ka^2B(ka) + \cdots\right]\varphi_0(k)\varphi_0(-k)\,, \end{aligned} \tag{4.9}$$

where $\operatorname{div}(\Lambda,k,a)$ indicates terms that are divergent when $\Lambda \to \infty$ and which are subtracted away. The dots denote terms that vanish when $\Lambda \to 0$. So we find that the correlation function is proportional to the coefficient B in (4.7), which in turn is the same as the reflection amplitude in [17].

$$\langle\mathcal{O}(k)\mathcal{O}(k')\rangle \sim \frac{\delta^2 S}{\delta\varphi_0(k)\delta\varphi_0(k')} \sim \delta(k+k')ka^2\ B(ka)\,. \tag{4.10}$$

Since the renormalization depends on the momentum it is not possible to go back to coordinate space in an unambiguous fashion. This is a reflection of the non-local nature of the theory in the ultraviolet. The situation is essentially the same as in [18]. With this prescription it is easy to see that we recover the $AdS_5 \times S^5$ results for $ka \ll 1$. The leading deviations from the AdS result are the same as the ones computed in [17] (we should replace $R^4 \to a^4$ in their formulas). From the point of view of the IR conformal field theory these corrections arise from a dimension 8 operator. In our context we can view this operator as arising from the expansion of the Born Infeld action when we have a B field.

To study the UV regime, we need to determine the value of $B(ka)$ at large k. As explained above, by $z \to z + i\pi/2$ in (4.5) this problem is converted into the problem of calculating the reflection coefficient for a particle with an energy greater than the potential barrier. This can be done by using the WKB approximation [19]. For $k^2a^2 \gg 1$, we find the following result:

$$\langle T_{01}(k) T_{01}(-k) \rangle \cong \exp\left[\, - c|k|a\right], \tag{4.11}$$

where

$$c = \sqrt{2} \int_0^{\pi/2} dx \, \sqrt{\cos(x)} = \frac{2}{\sqrt{\pi}} \Gamma(3/4)^2 \cong 1.69\,. \tag{4.12}$$

Thus the correlator vanishes in the UV limit $k^2 \to \infty$. This is a drastic change with respect to the commutative case. We should remember, however, that we have performed a momentum dependent renormalization, so there is some ambiguity in this result. We only expect ratios of three (or higher n) point functions to two-point functions to be unambiguous.

Let us now discuss what we expect from the point of view of field theory. The arguments of [9] would naively indicate that all calculations in the large-N limit should reduce trivially to the commutative counterparts. We should remember that they had obtained some phases for the external momenta. Defining operators involves taking a particular superposition of bare operators. It is not easy to find the gauge invariant operator in the non-commutative Yang-Mills theory representing $T_{01}(k)$ (the Fourier transform of Tr $F * F$ is not gauge invariant). In the presence of b we would take a different supperposition of operators and therefore the results could change.

5. Wilson loops

Let us first consider the case when only $B_{23} \neq 0$. The near-horizon geometry is given by (2.7). A single string stretching from the horizon to the boundary has the same energy as in the AdS case, i.e. the energy grows with the cutoff in the same manner. Now suppose we bring two of these strings close to each other, at different points in the x_2, x_3 plane. Naively one would imagine that they connect and give rise to some

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potential. This is not what happens. The Nambu-Goto action for a single string with $x_0 = \tau$, $u = \sigma$, $x^i(u)$, $i = 2, 3$, is given by

$$S = \frac{R^2}{2\pi} \int dx_0 du \sqrt{1 + (\partial_u x^i)^2 u^4 \hat{h}}\,. \tag{5.1}$$

Expanding (5.1) for small fluctuations x^i at large u we get $S \sim \int dx_0 du (\partial_u x^i)^2$. This implies that we cannot fix the position of the string at infinity since any small perturbation implies that x^i grows linearly with u. We can however specify the slope k^i which the string has as it approaches infinity, $x^i \sim k^i u$. We will see below that, in a loose sense, we can interpret k to be related to some kind of Fourier transform of the Wilson loop. The Wilson loop operator, which starts extended along the time direction and seems localized in x^2, x^3, is now replaced by a new operator which is characterized by the "momentum" k in the x_2, x_3 directions. In this way we can calculate the potential energy between a quark with "momentum" k with an anti-quark with "momentum" k'. This will vanish unless $k^i = -k'^i$. This can be seen by taking the action (5.1) and noticing that, interpreting u as "time" then the canonical momenta conjugate to x^i are conserved. So we can choose k to point in the direction 3. Denoting by u_0 the coordinate of closest approach to the horizon we find that

$$x^3(u) = \int_{u_0}^{u} du' \frac{u_0^2 (1 + a^4 u'^4)}{u'^2 \sqrt{u'^4 - u_0^4}}\,. \tag{5.2}$$

From the large u behavior we find that $k = u_0^2 a^4$. We can calculate the energy, which is divergent. The divergent piece depends on k, as we found for the correlation functions. We subtract it and we get a finite answer which is

$$E = -\frac{R^2 \sqrt{2\pi}}{\Gamma(1/4)^2} \sqrt{1 + \frac{k^2}{a^4}} \frac{\sqrt{k}}{a^2}\,. \tag{5.3}$$

In order to make contact with the standard AdS expression notice that for small k then u_0 will be very small ($a u_0 \ll 1$). Looking at (5.2) we see that we get a large region in the radial coordinate where we can ignore the term proportional to $a^4 u'^4$. In that case we recover the AdS expression for the coordinate x, and in particular we find that the separation is $L \sim 1/u_0 \sim a^2/\sqrt{k}$. Substituting in (5.3) we recover the AdS expression, $E \sim 1/L$. We did not get this relationship between distance and "momentum" by performing a Fourier transformation, and it is not standard, that is the reason for the quotation marks in "momentum".

Now consider a string configuration in the geometry (2.9) (in the particular case $a = a'$) of the form

$$x_1 = \tau\,, \qquad x_2 = \sigma\,, \qquad u = u(\sigma)\,, \qquad x_0 = x_3 = 0\,, \tag{5.4}$$

which is placed at some given angle in the 5-sphere. Using eqs. (2.7) and (5.4), the action takes the form

$$S = \frac{R^2}{2\pi} \int d\tau d\sigma \sqrt{\hat{h}} \sqrt{(\partial_\sigma u)^2 + u^4 \hat{h}}\,, \qquad \hat{h} = \frac{1}{1 + a^4 u^4}\,. \tag{5.5}$$

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The solution that minimizes the action is given by

$$\frac{u^4\hat{h}^{3/2}}{\sqrt{(\partial_\sigma u)^2+u^4\hat{h}}}=\frac{e}{a^2}=\text{const.}\,, \tag{5.6}$$

or

$$e\partial_\sigma u=u^2\sqrt{\hat{h}}\sqrt{a^4u^4\hat{h}^2-e^2}\,, \tag{5.7}$$

where e is an integration constant. The equation (5.7) is symmetric under $u \to 1/(a^2u)$ since the string metric has this symmetry. We have solutions for $0 < e \leq 1/2$. If $e = 1/2$ the solution is a straight worldsheet sitting at $u = a$, which is the maximum of the potential that the string sees. For $0 < e < 1/2$ the solution oscillates between $u_{\min}$ and $u_{\max} = 1/(a^2u_{\min})$ as we move in $x_2 = \sigma$. $u_{\max}, u_{\min}$ are solutions of $\partial_\sigma u = 0$. The strings never get to the boundary. If we compactify the x_1, x_2 directions these solutions describe finite action worldsheet instantons. In this case e will be quantized so that we have an integral number of oscillations in the x_2 circle. When $e \ll 1/2$ we get a string configuration which in the region $u \ll 1/a$ looks like the Wilson loops of AdS_5.

6. Generalizations

6.1 D1-D5 system with a B field

A D1-D5 system with a B field can be obtained likewise by U-duality. We start with the usual D1-D5 system in euclidean space, with world-volume coordinates x_0, x_1, and make a T-duality transformation in the direction x_1, obtaining a D0-D4 bound state with an extra translational isometry in x_1. Then we redefine coordinates as follows,

$$x_0=x_0'\cos\theta+x_1'\sin\theta\,,\qquad \tilde{x}_1=-x_0'\sin\theta+x_1'\cos\theta\,. \tag{6.1}$$

By applying a T-duality transformation in the x_1' direction, we find (restoring the labels x_0, x_1 for the 2-plane coordinates and performing a gauge transformations that changes the asymptotic value of B)

$$\begin{aligned}
ds^2_{str} &= (f_1f_5)^{-1/2}h(dx_0^2+dx_1^2)+f_1^{1/2}f_5^{1/2}(dr^2+r^2d\Omega_3^2)+\\
&\quad +f_1^{1/2}f_5^{-1/2}dy_ndy_n\,,\qquad n=1,2,3,4\,,\\
h^{-1} &= (f_1f_5)^{-1}\sin^2\theta+\cos^2\theta\,,\qquad f_{1,5}=1+\frac{\alpha' R_{1,5}^2}{r^2}\,,\\
e^{2\phi} &= g^2\frac{f_1}{f_5}h\,,\\
\chi &= i\frac{1}{g}\sin\theta f_1^{-1}\,,\qquad A_{y_1y_2y_3y_4}=i\frac{1}{g}\sin\theta f_5^{-1}\,,\\
B_{01} &= \frac{\sin\theta}{\cos\theta}\,(f_1f_5)^{-1}h\,,\\
dA_2 &= \frac{1}{g}\cos\theta(i\;d(hf_1^{-1})\wedge dx_0\wedge dx_1+2R_5^2\epsilon_3)\,,
\end{aligned} \tag{6.2}$$

where ϵ_3 is the volume element of a unit radius 3-sphere. By a similar rescaling of parameters as in (2.6)one obtains the near-horizon geometry

$$\begin{aligned}
ds^2_{str} &= \alpha' R^2 \left[u^2 \hat{h}(dx_0^2 + dx_1^2) + \frac{du^2}{u^2} + d\Omega_3^2 \right] + \frac{R_1}{R_5} dy_n dy_n \,, \\
e^{2\phi} &= \hat{g}^2 \hat{h} \,, \qquad\qquad \hat{h} = \frac{1}{1 + a^4 u^4} \,, \\
B_{01} &= B_\infty \frac{a^4 u^4}{1 + a^4 u^4} \,, \qquad\qquad B_\infty = \alpha' \frac{R^2}{a^2} \,, \\
\chi &= \frac{i}{\hat{g}} a^2 u^2 \,, \qquad\qquad A_{y_1 y_2 y_3 y_4} = \frac{i}{\hat{g}} \frac{R_1^2}{R_5^2} a^2 u^2 \,, \\
dA_2 &= \frac{1}{\hat{g}} R_1 R_5 \Big(i\alpha' d(u^2 \hat{h}) \wedge dx_0 \wedge dx_1 + 2\epsilon_3 \Big) \,, \\
R^4 &= R_1^2 R_5^2 \,, \qquad\qquad \frac{a^2}{R^2} = \alpha' \tan\theta \,, \qquad \hat{g} = \frac{g}{\alpha'} \frac{a^2}{R_5^2} \,.
\end{aligned} \tag{6.3}$$

Near $u = 0$ and near $u = \infty$, the metric approaches $AdS_3 \times S^3 \times T^4$. Although the string frame metric is symmetric under $u \to 1/(a^2 u)$, the dilaton and gauge fields behave differently in the UV region $u = \infty$. All these solutions, (6.2) and (6.3), preserve 8 supersymmetries.

A D1-D5 configuration with B components in two five-brane directions y_1, y_2 which are transverse to the D1-brane can be obtained as follows. We apply a T-duality in y_2, do a rotation in the y_1, y_2 plane, a T-duality back on y_2 and a gauge transformation to change the asymptotic value of B. The solution is given by

$$\begin{aligned}
ds^2_{str} &= f_1^{-1/2} f_5^{-1/2} \big(-dx_0^2 + dx_1^2 \big) + f_1^{1/2} f_5^{1/2} (dr^2 + r^2 d\Omega_3^2) + \\
&\quad + f_1^{1/2} f_5^{-1/2} \left[h(dy_1^2 + dy_2^2) + dy_3^2 + dy_4^2 \right] \\
e^{2\phi} &= g^2 \frac{f_1}{f_5} h \,, \qquad B_{y_1 y_2} = \frac{\sin\theta}{\cos\theta} \frac{f_1}{f_5} h \,, \\
h^{-1} &= \cos^2\theta + \sin^2\theta \frac{f_1}{f_5} \,, \\
dA_2 &= \frac{1}{g} \cos\theta (df_1^{-1} \wedge dx_0 \wedge dx_1 + 2R_5^2 \epsilon_3) \,, \\
F &= G + *_{10} G \,, \qquad G = \sin\theta h \frac{f_1}{f_5}\, dy_1 \wedge dy_2 \wedge \Big(df_1^{-1} \wedge dx_0 \wedge dx_1 + 2R_5^2 \epsilon_3 \Big) \,.
\end{aligned} \tag{6.4}$$

General Dp brane solutions with a B-field can be constructed in the same way that (2.1) was obtained. The resulting background is given by

$$\begin{aligned}
ds^2_{str} &= f^{-1/2} [-dx_0^2 + dx_1^2 + h(dx_2^2 + dx_3^2) + \cdots + dx_p^2] + f^{1/2} (dr^2 + r^2 d\Omega_{8-p}^2) \,, \\
f &= 1 + \frac{R^{7-p}}{r^{7-p}} \,, \qquad h^{-1} = \sin^2\theta f^{-1} + \cos^2\theta \,, \\
B_{23} &= \frac{\sin\theta}{\cos\theta} f^{-1} h \,, \\
e^{2\phi} &= g^2 f^{\frac{3-p}{2}} h \,.
\end{aligned} \tag{6.5}$$

And we also have RR fields excited corresponding to Dp brane charges and D$(p-2)$ brane charge densities. It is interesting to note that the string coupling decreases faster at infinity than in the commutative case. This suggests that maybe it is possible to define the non-commutative version of the D6 brane when we have two B fields.[4] By taking the large-N limit of this field theory we would describe M-theory on T^6 according to [20]. From our count of degrees of freedom above it looks like we will have problems similar to the ones appearing in the commutative case. In particular, we can calculate the entropy which gives the same result as in the commutative case. The entropy goes as $S \sim E^{3/2}$, which implies that the system has a negative specific heat. This does not obviously imply non-decoupling from the bulk and a more detailed analysis is necessary.

6.2 M5 brane with C-field

One can similarly find an M5 brane solution with a C-field. In this case we get

$$
\begin{aligned}
ds_{11}^2 &= k^{1/3} f^{1/3} \left[\frac{1}{f}(- dx_0^2 + dx_1^2 + dx_2^2) + \frac{1}{k}(dx_3^2 + dx_4^2 + dx_5^2) + dr^2 + r^2 d\Omega_4^2 \right] , \\
f &= 1 + \frac{R^3}{r^3} , \\
k &= \sin^2\theta + \cos^2\theta \; f , \\
dC_3 &= \sin\theta \; df^{-1} \wedge dx_0 \wedge dx_1 \wedge dx_2 + \cos\theta 3 R^3 \epsilon_4 - \\
&\quad - 6 \tan\theta d(k^{-1}) \wedge dx_3 \wedge dx_4 \wedge dx_5 \,.
\end{aligned} \tag{6.6}
$$

This solution appeared in [7] (eq. (2.26)) and it was interpreted as a 2-brane lying within a 5-brane. One can similarly introduce a decoupling limit in such a way that we obtain the solution (6.6) but with $f \to R^3/r^3$. Notice that the fivebrane charge is given by $\pi N = R^3/\cos\theta$. The DLCQ definition of this theory was considered in [10].

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